

# Pre-Calculus

# Review for Chapter 5 Exam

Use the fundamental identities to find the value of the trigonometric function.

Find  $\sec \theta$  if  $\cot \theta = -\sqrt{5}$  and  $\cos \theta < 0$

$$\sec \theta = \frac{r}{x}$$

$$\cot \theta = \frac{y}{x} = \frac{\sqrt{5}}{1}$$

$$x^2 + y^2 = r^2$$

$$(1)^2 + (\sqrt{5})^2 = r^2$$

$$\pm \sqrt{6} = r$$

$$\sec \theta = \frac{r}{x} = \frac{\sqrt{6}}{1} = \sqrt{6} \quad \text{or} \quad \sec \theta = \frac{r}{x} = \frac{-\sqrt{6}}{1} = -\sqrt{6}$$

Use basic identities to simplify the expression.

$$\tan \theta \csc \theta \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta} \cdot \cos \theta = 1$$

Simplify the expression.

$$\tan x \cos x - \cos\left(\frac{\pi}{2} - x\right) + \sin x$$

$$\frac{\sin x}{\cos x} \cdot \cos x - \sin x + \sin x = \sin x$$

Find all solutions in the interval  $[0, 2\pi)$ .

$$\sin^2 x + 2 \sin x + 1 = 0$$

$$(\sin x + 1)^2 = 0$$

$$\sin x = -1$$

$$x = 3\pi/2$$

$$\sin x = \sin 2x$$

$$\sin x - \sin 2x = 0$$

$$\sin x - 2 \sin x \cos x = 0$$

$$\sin x (1 - 2 \cos x) = 0$$

$$\sin x = 0$$

$$x = 0, \pi$$

$$1 - 2 \cos x = 0$$

$$\cos x = 1/2$$

$$x = \pi/3, 5\pi/3$$

Find an exact value.

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$\cos\left(-\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{3}\right) \cos\left(\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{3}\right) \sin\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{3}}{2}\right) \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

Write the expression as the sine, cosine, or tangent of an angle.

$$\cos \frac{\pi}{3} \cos \frac{\pi}{5} + \sin \frac{\pi}{3} \sin \frac{\pi}{5}$$

$$\sin \frac{\pi}{3} \cos \frac{\pi}{5} + \cos \frac{\pi}{3} \sin \frac{\pi}{5}$$

$$\cos\left(\frac{\pi}{3} - \frac{\pi}{5}\right) = \cos \frac{2\pi}{15}$$

$$\sin\left(\frac{\pi}{3} + \frac{\pi}{5}\right) = \sin \frac{8\pi}{15}$$

Find the exact value by using a half-angle identity.

$$\cos 22.5^\circ = \cos \left(\frac{45^\circ}{2}\right) = \pm \sqrt{\frac{1 + \cos 45^\circ}{2}}$$

$$= \pm \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \pm \sqrt{\frac{\frac{2}{2} + \frac{\sqrt{2}}{2}}{2}} = \pm \sqrt{\frac{2 + \sqrt{2}}{4}}$$

$$= \pm \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\frac{\sqrt{2 + \sqrt{2}}}{2}$$

skip

Solve the triangles.

$$\begin{aligned} a &= 5.17 \\ c &= 8.99 \\ \angle C &= 94^\circ \end{aligned}$$

$$A = 35^\circ, B = 51^\circ, b = 7$$

$$\frac{\sin 51^\circ}{7} = \frac{\sin 35^\circ}{a}$$

$$\frac{\sin 94^\circ}{c} = \frac{\sin 51^\circ}{7}$$

$$\begin{aligned} \angle B &= 43.7^\circ \\ \angle A &= 33.3^\circ \\ a &= 17.47 \end{aligned}$$

$$C = 103^\circ, a = 31, b = 22$$

$$\frac{\sin 103^\circ}{31} = \frac{\sin B}{22}$$

$$\frac{\sin 33.3^\circ}{a} = \frac{\sin 103^\circ}{31}$$

State whether the given measurements determine zero, one, or two triangles.

$A = 36^\circ, a = 2, b = 7$

No triangles

$$7 \sin 36^\circ = 4.1$$

$$2 < 4.1$$

$B = 82^\circ, b = 17, c = 15$

$b > c$

One solution.

$C = 36^\circ, a = 17, c = 16$

$$17 \sin 36^\circ = 9.99$$

$$17 > 16 > 9.99$$

Two solutions

Two triangles can be formed using the given measurements. Solve both triangles.

$a = 11, B = 57^\circ, b = 10$

$$\frac{\sin 57^\circ}{10} = \frac{\sin A}{11}$$

$$\frac{\sin 55.7^\circ}{c} = \frac{\sin 57^\circ}{10}$$

$$\angle A_1 = 67.3^\circ, c_1 = 9.85$$

$$\angle C_1 = 55.7^\circ$$

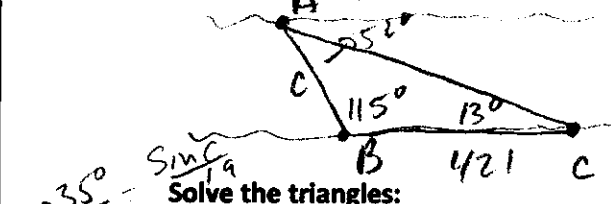
$$\angle A_2 = 112.7^\circ$$

$$\angle C_2 = 10.3^\circ$$

$$c_2 = 2.13$$

$$\frac{\sin 10.3^\circ}{c} = \frac{\sin 57^\circ}{10}$$

Solve: To find the distance AB across a river, a distance BC of 421 m is laid off on one side of the river. It is found that  $B = 115^\circ$  and  $C = 13^\circ$ . Find AB.



$$\frac{\sin 13^\circ}{AB} = \frac{\sin 52^\circ}{421}$$

$$AB = 120.18 \text{ m}$$

Solve the triangles:

$B = 35^\circ, a = 43, c = 19$

$$b^2 = 43^2 + 19^2 - 2(43)(19)\cos 35^\circ$$

$$b^2 = 871.51$$

$$b = 29.52$$

$$\angle C = 21.7^\circ$$

$$\angle A = 123.3^\circ$$

Find the area.

$A = 55^\circ, c = 21, b = 14$

$$\frac{1}{2} \cdot 14 \cdot 21 \cdot \sin 55^\circ = \frac{240.83}{2} = 120.42$$

$a = 3, b = 7, c = 6$

$$3^2 = 7^2 + 6^2 - 2(7)(6)\cos A$$

$$9 = 85 - 84\cos A$$

$$-76 = -84\cos A$$

$$.90476 = \cos A$$

$$25.2^\circ = \angle A, \angle C = 58.4^\circ$$

$$\angle B = 96.4^\circ$$

$$\frac{\sin 25.2^\circ}{3} = \frac{\sin C}{6}$$

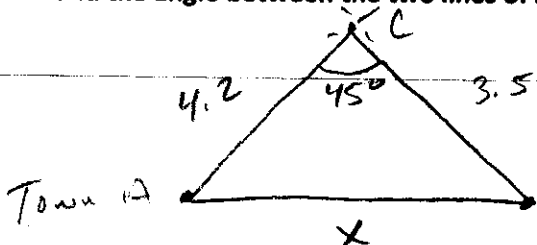
Decide whether a triangle can be formed with the given side lengths. If so, use Heron's formula to find the area of the triangle.  $a = 5, b = 9, c = 7$

$$s = 10.5$$

$$A = \sqrt{10.5(5.5)(1.5)(3.5)} = 17.1$$

Yes  $7+5 > 9$

To find the distance between two small towns, an electronic distance measuring (EDM) instrument is placed on a hill from which both towns are visible. If the distance from the EDM to the towns is 4.2 miles and 3.5 miles and the angle between the two lines of sight is  $45^\circ$ , what is the distance between the towns?



$$X^2 = (4.2)^2 + (3.5)^2 - 2(4.2)(3.5)\cos 45^\circ$$

$$X^2 = 9.10$$

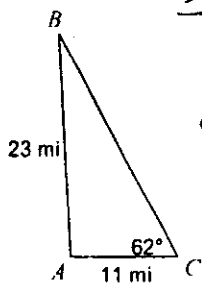
$$X = 3.02 \text{ miles}$$

## Law of Sines and Cosines

Date \_\_\_\_\_ Period \_\_\_\_\_

Solve each triangle. Round your answers to the nearest tenth.

1)



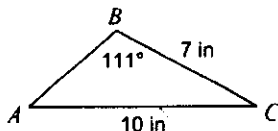
$$\frac{\sin 62}{23} = \frac{\sin B}{11}$$

$$\angle A = 93^\circ$$

$$\angle B = 25^\circ$$

$$a = 26 \text{ mi}$$

2)



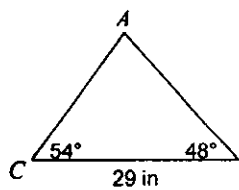
$$\angle C = 28.2^\circ$$

$$\angle A = 40.8^\circ$$

$$c = 5.1 \text{ in}$$

4  
24  
248  
32

3)

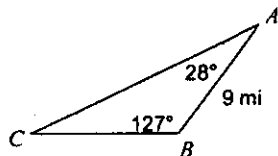


$$\angle A = 78^\circ$$

$$b = 22 \text{ in}$$

$$c = 24 \text{ in}$$

4)

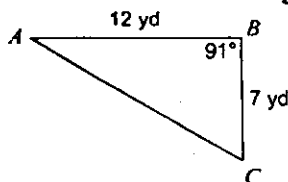


$$\angle C = 25^\circ$$

$$b = 17 \text{ mi}$$

$$a = 10 \text{ mi}$$

5)

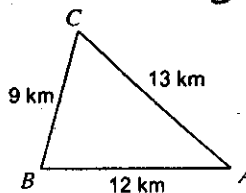


$$\angle C = 59^\circ$$

$$\angle A = 30^\circ$$

$$b = 14 \text{ yd}$$

6)

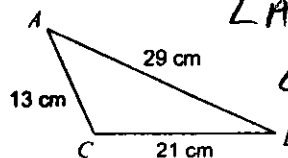


$$\angle C = 63^\circ$$

$$\angle A = 42^\circ$$

$$\angle B = 75^\circ$$

7)

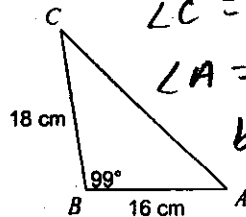


$$\angle A = 41^\circ$$

$$\angle B = 24^\circ$$

$$\angle C = 115^\circ$$

8)



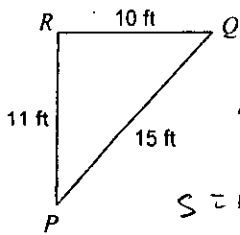
$$\angle C = 37.7^\circ$$

$$\angle A = 43.3^\circ$$

$$b = 25.9 \text{ cm}$$

Find the area of each triangle to the nearest tenth.

9)

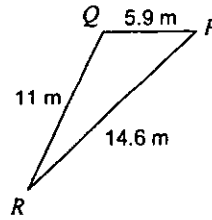


$$55 \text{ ft}^2$$

$$s = 18$$

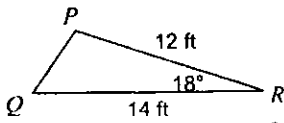
$$A = \sqrt{18 \cdot 3 \cdot 8 \cdot 7} \approx 54.99 \approx 55 \text{ ft}^2$$

10)



$$29.1 \text{ m}^2$$

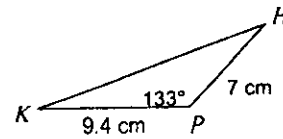
11)



$$26 \text{ ft}^2$$

$$\frac{1}{2} \cdot 14 \cdot 12 \cdot \sin 18^\circ \approx 26 \text{ ft}^2$$

12)



$$24.1 \text{ cm}^2$$